Quick 2

Formalization and Resolution (6 points)

In the province of *Groland*, the new laws about retirement are discussed everywhere, as can be heard in that debate between *Guigolaine Plutanfiard* and *Fifigrot Vichume*, both *Mufflins* inhabitants.

- Guigolaine says: "If you are 62 at least, then you are allowed to take retirement."
- Fifigrot answers: "You can take retirement if and only if you are 62 at least and you've paid your fees, or you've reached pivotal age."
- Guigolaine again: "If you've reached pivotal age, then you are 62 at least."
- Then Fifigrot concludes: "Thus, if you've paid your fees but you're not 62 at least, then you cannot take retirement."

1. Formalize the three statements and the conclusion.

   You will use the following propositional variables: L means "to be 62 at least", R means "to be allowed to take retirement",F means "to have paid your fees" and P means "to have reached pivotal age".

   - $H_1 : L \Rightarrow R$;
   - $H_2 : R \Leftrightarrow (LF + P)$;
   - $H_3 : P \Rightarrow L$;
   - $C : (F\bar{L}) \Rightarrow \bar{R}$.

2. Transform these three statements and the negation of the conclusion into an equivalent set of clauses.

   $H_1 : \bar{L} + R$;

H_2:

$$
\begin{aligned}
R \Leftrightarrow (LF+P) &\equiv (R \Rightarrow (LF+P))((LF+P) \Rightarrow R)\\
&\equiv (\bar{R}+(LF+P))(\overline{(LF+P)}+R)\\
&\equiv (\bar{R}+LF+P)((\overline{LF}\bar{P})+R)\\
&\equiv ((\bar{R}+L)(\bar{R}+F)+P)(((\bar{L}+\bar{F})\bar{P})+R)\\
&\equiv ((\bar{R}+L+P)(\bar{R}+F+P))((\bar{L}+\bar{F}+R)(\bar{P}+R))\\
&\equiv (\bar{R}+L+P)(\bar{R}+F+P)(\bar{L}+\bar{F}+R)(\bar{P}+R)
\end{aligned}
$$

H_3: $\bar{P}+L$;

$\bar{C}$:

$$
\begin{aligned}
\overline{(F\bar{L}) \Rightarrow \bar{R}} &\equiv \overline{\overline{(F\bar{L})}+\bar{R}}\\
&\equiv \overline{\bar{F}+L+\bar{R}}\\
&\equiv \overline{\bar{F}+L}R\\
&\equiv F\bar{L}R
\end{aligned}
$$

We obtain the following set of clauses:

$\Gamma = \{\bar{L}+R, \bar{R}+L+P, \bar{R}+F+P, \bar{L}+\bar{F}+R, \bar{P}+R, \bar{P}+L, F, \bar{L}, R\}$

3. Prove, ***by resolution***, that the conclusion of Fifigrot is correctly deduced from the three first statements. To prove that the reasoning of Fifigrot is correct, we prove that the set of clauses Γ is unsatisfiable.

1	$\bar{L}$	`Hyp`
2	$\bar{P}+L$	`Hyp`
3	$\bar{P}$	`Res. 1, 2`
4	$\bar{R}+L+P$	`Hyp`
5	$\bar{R}+P$	`Res 1, 4`
6	$\bar{R}$	`Res 3, 5`
7	R	`Hyp`
8	$\perp$	`Res 6, 7`

Therefore the reasoning of Fifgrot is correct.

DPLL (4 points)

Use the `DPLL` algorithm to determine whether the following set of clauses is satisfiable or not. Give a tree-like execution trace of the algorithm, where every step should be clearly labeled by the rule that you used. You should interpret clearly the obtained result when the algorithm terminates and give a model if applicable.

$$\{\ a+b+c+d+e+f,\ \overline{a}+b,\ \overline{b}+a,\ \overline{c}+d,\ \overline{d}+c,\ \overline{b}+\overline{c},\ \overline{b}+c,\ b+\overline{c},\ \overline{e},\ \overline{f}\ \}$$

$$\begin{tikzcd}
a+b+c+d+e+f,\overline{a}+b,\overline{b}+a,\overline{c}+d,\overline{d}+c,\overline{b}+\overline{c},\overline{b}+c,b+\overline{c},\overline{e},\overline{f} \arrow[d, "\text{UR } e=0,\ f=0"'] & \\
a+b+c+d,\overline{a}+b,\overline{b}+a,\overline{c}+d,\overline{d}+c,\overline{b}+\overline{c},\overline{b}+c,b+\overline{c} \arrow[d, "a=0"'] \arrow[dr, "a=1"] & \\
b+c+d,\overline{b},\overline{c}+d,\overline{d}+c,\overline{b}+\overline{c},\overline{b}+c,b+\overline{c} \arrow[d, "\text{RE}"'] & b,\overline{c}+d,\overline{d}+c,\overline{b}+\overline{c},\overline{b}+c,b+\overline{c} \arrow[d, "\text{RE}"'] \\
b+c+d,\overline{b},\overline{c}+d,\overline{d}+c,b+\overline{c} \arrow[d, "\text{UR } b=0"'] & b,\overline{c}+d,\overline{d}+c,\overline{b}+\overline{c},\overline{b}+c \arrow[d, "\text{UR } b=1"'] \\
c+d,\overline{c}+d,\overline{d}+c,\overline{c} \arrow[d, "\text{UR } c=0"'] & \overline{c}+d,\overline{d}+c,\overline{c},c \arrow[d, "\text{UR}"'] \\
d,\overline{d} \arrow[d, "\text{UR}"'] & \bot \\
\bot &
\end{tikzcd}$$

This set of clauses is unsatisfiable, hence it does not have any model.