

Quick 1° - INF432

(30 minutes) - Groupe MIN-int

Exercice 1 : Priorities

(3 points)

Let A be the following formula :

$$\neg a \Rightarrow b \wedge c \Leftrightarrow \neg c \wedge a$$

- Give the corresponding strict formula.
- Give the corresponding formula in boolean notation.
- Give the corresponding tree structure.

Remind : Booleans connectors by decreasing order of priority : $\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$.

Exercice 2 : Consequence

(3 points)

Tonight there will be a big concert in Grenoble, and there could be three bands playing : the Pure Players, the Quantic Quintet and the Roaming Rebels.

- If the Pure Players are not playing, then the Quantic Quintet will play.
- If the Quantic Quintet play, then all three bands will play.
- If the Roaming Rebels play, then the Pure Players or the Quantic Quintet, at least one of them, will not play.

Questions :

- Model the facts
- Can we conclude from these 3 facts that the Pure Players will play tonight ? Justify.

Exercice 3 : Induction proof

(4 points)

For all formulae A and B , we have :

- $\top \equiv \perp \Rightarrow \perp$,
- $\neg A \equiv A \Rightarrow \perp$,
- $A \vee B \equiv \neg A \Rightarrow B$,
- $A \wedge B \equiv \neg(\neg A \vee \neg B)$,
- $A \Leftrightarrow B \equiv (A \Rightarrow B) \wedge (B \Rightarrow A)$.

Show by induction over the size of a formula that any formula¹ can be written using only variables, \perp and \Rightarrow . Emphaze the base case, the induction hypothesis and the induction step.

Remind : The size of a formula A , denoted $|A|$, is recursively defined by :

- $|\top| = 0$ and $|\perp| = 0$.
- If A is a variable then $|A| = 0$.
- $|\neg A| = 1 + |A|$.
- $|(A \circ B)| = |A| + |B| + 1$, if \circ is one of the operations $\vee, \wedge, \Rightarrow, \Leftrightarrow$.

1. Note that unnecessary parenthesis are omitted.